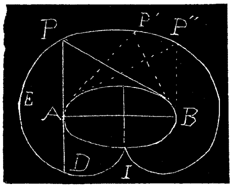
a corollary of Graves' Theorem. This was the interpretation according to which Dr. Haskell made his solution and which was inadvertently published in the MONTHLY, Vol. V, No. 4, page 111.



The problem as proposed is quite different and is not easy to solve, perhaps impossible.

The area grazed over consists of three parts :

(1) The part $ABP''P$—the area of this part is not difficult to find as the arc $PP'P''$ is the arc of a confocal ellipse; (2) two times the part $PEDAP$,—this part is generated by point, P, constrained to move under the composition of circular and elliptic evolutary motion ; and (3) two times the part $ADIA$, which is between the evolute of the elliptic field, the field, and the radius vector AD.

The part $PEDAP$ is the difficult part of the area to compute.

If any of our contributors will furnish a correct and complete solution of this problem it will be published in the next issue of the MONTHLY.

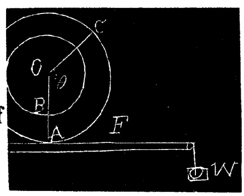
MECHANICS.

98. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

A spool, with light thread wound around, is placed upon a rough table so that the thread will emerge from beneath the spool. The thread is passed over a smooth pulley at end of table and a weight attached, the pulley being so adjusted that thread is parallel to surface of table. If friction between spool and table is sufficient to prevent slipping, determine motion of spool and weight. [From problems in Mechanics at Harvard University.]

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let T=tension on thread, x=distance spool's center moves, y=distance weight W moves, θ=angle AOC through which spool turns, $OA=a$, OB the radius of the thread=b.



Then $x=a\theta$, $y=(a-b)\theta$.

The forces acting on the spool are friction, F; tension, T; and reaction, R, perpendicular to the table.

Let m=mass of spool and thread, m_1=mass of weight. Then

$$m\frac{d^2x}{dt^2}=T-F,\ mk^2\frac{d^2\theta}{dt^2}=Fa..(1,2),\ \text{for spool};\ m_1\frac{d^2y}{dt^2}=T-m_1g..(3),\ \text{for weight.}$$

Eliminating F between (1, 2) we get

$$ma\frac{d^2x}{dt^2}+mk^2\frac{d^2\theta}{dt^2}=T....(4)\ ;\ \text{but}\ \frac{d^2x}{dt^2}=a\frac{d^2\theta}{dt^2},\ \frac{d^2y}{dt^2}=(a-b)\frac{d^2\theta}{dt^2}.$$

$\therefore\ m(a^2+k^2)\dfrac{d^2\theta}{dt^2}=T\ldots(5)$, for spool; $m_1(a-b)\dfrac{d^2\theta}{dt^2}=T-m_1g\,..(6)$, for weight.

Eliminating T between (5) and (6), we get

$$\frac{d^2\theta}{dt^2}=\frac{m_1g}{m(a^2+k^2)+m_1(b-a)}.\qquad \therefore\ T=\frac{mm_1(a^2+k^2)g}{m(a^2+k^2)+m_1(b-a)}.$$

$$\therefore\ m(a^2+k^2)\frac{d^2x}{dt^2}=\frac{amm_1(a^2+k^2)g}{m(a^2+k^2)+m_1(b-a)}$$

$$\text{or } \frac{d^2x}{dt^2}=\frac{am_1g}{m(a^2+k^2)+m_1(b-a)}$$

gives the acceleration of the spool. Also from (3),

$$m_1\frac{d^2y}{dt^2}+m_1g=\frac{mm_1(a^2+k^2)g}{m(a^2+k^2)+m_1(b-a)}.\quad \therefore\ \frac{d^2y}{dt^2}=-\frac{m_1(b-a)g}{m(a^2+k^2)+m_1(b-a)},$$

gives acceleration of weight.

$\therefore$ If $b=a$, the weight remains at rest; if $a>b$, the weight ascends.

II. **Solution by L. R. INGERSOLL, Student in Colorado College, Colorado Springs, Col., and the PROPOSER.**

Let a be the radius of inner portion of spool, b radius of end. Then for rotation of spool about its center C we have the equation

$$\tfrac{1}{2}Mb^2\frac{d^2\theta}{dt^2}=Fb-mga\ldots(1).$$

For motion of center of spool, the equation is

$$M\frac{d^2x}{dt^2}=mg-F\ldots(2).$$

Eliminating F and remembering that $x=b\theta$, equations (1) and (2) become

$$\frac{d^2\theta}{dt^2}=\frac{2mg(b-a)}{3b^2M}\ldots(3),\text{ and }\frac{d^2x}{dt^2}=\frac{2mg(b-a)}{3bM}\ldots(4).$$

If $b>a$, the spool will roll towards weight with a constant angular acceleration, and its center will move with a constant acceleration b times its angular acceleration. But for same reason the string will be wound up with an acceleration a times its angular acceleration, and hence weight will descend with an acceleration $(b-a)$ times the angular acceleration.

If $b=a$, the spool will move towards or from the weight with constant velocity, and the weight will remain stationary.

If $b<a$, the spool will roll from the weight with constant angular acceler-

ation, and weight will descend with an acceleration $a-b$ times the angular acceleration.

99. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

In a triangle ABC, base$=b$, area$=\triangle$, the principal moments of inertia at the centroid are $\frac{1}{72}m[a^2+b^2+c^2\pm\sqrt{(a^4+b^4+c^4-a^2b^2-a^2c^2-b^2c^2)}]$ and the principal axes at this point make with the base AC an angle θ given by

$$\tan2\theta=\frac{4(c^2-a^2)\triangle}{(a^2-c^2)^2-b^2(a^2+c^2)+2b^4}.$$

Solution by the PROPOSER.

Let O be the centroid and transform from the rectangular axes Ox, Oy, to the oblique axes Ox, OB.

Also let $(b/6AD)(2AD-3y)=x'$.

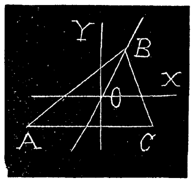
$$\text{Then } \Sigma mxy=\rho\sin^2D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}(x+y\cos D)ydydx=\tfrac{1}{36}b\rho AD^3\sin^2D\cos D$$

$$=\tfrac{1}{18}mAD^2\sin D\cos D.$$

$$\Sigma mx^2=\rho\sin D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}(x+y\cos D)^2dydx=\tfrac{1}{72}m(3b^2+4AD^2\cos^2D).$$

$$\Sigma my^2=\rho\sin^3D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}y^2dydx=\tfrac{1}{18}mAD^2\sin^2D.$$

$$\therefore\ \tan2\theta=\frac{4AD^2\sin2D}{3b^2+4AD^2\cos2D}.$$



But $\sin D=a\sin C/AD$, $\cos D=\dfrac{4AD^2+b^2-4a^2}{4b.AD}$.

$$\sin2D=\frac{a\sin C(4AD^2+b^2-4a^2}{2b.AD^2}=\frac{2(c^2-a^2)\triangle}{b^2.AD^2}.$$

$$\cos2D=\frac{AD^2-2a^2\sin^2C}{AD^2}=\frac{2(a^2-c^2)-2b^2(a^2+c^2)+b^4}{4b^2.AD^2}.$$

$$\therefore\ \tan2\theta=\frac{4(c^2-a^2)\triangle}{(a^2-c^2)^2-b^2(a^2+c^2)+2b^4}.$$

$$A\cos^2\theta+B\sin^2\theta=\tfrac{1}{72}m(3b^2+4AD^2\cos^2D).$$